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Disoriented Chiral Condensate and Strong Electromagnetic Fields

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Abstract

We discuss the effect of strong electromagnetic fields on chiral orientation in the framework of the linear σ model. Based on lessons we learn from computation of the effective potential at one loop, we argue that the chiral U(1) anomaly dominates chiral disorientation driven by electromagnetism. We show that the anomaly effect induces a quasi-instantaneous "kick" to field configurations along the π^0 direction in relativistic heavy ion collisions. We discuss the possibility that a "kick", even though small in magnitude, can have a substantial effect on the formation of chirally misaligned domains.

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There has recently been considerable interest in the possibility of the formation of a "chirally misaligned" domain of space-time as a result of high energy collisions among protons or heavy nuclei [1–7]. These domains have been called "disoriented chiral condensates" (DCC), because they can be formally described as localized, coherent excitations of the pion fields corresponding to a local rotation of the chiral order parameter of the QCD vacuum. Numerical simulations [4–6] have shown that such a state can be created spontaneously under circumstances where the chiral symmetry of the QCD vacuum has been temporarily restored due to a high temperature and then gets broken again during the expansion and cooling process.

Because heavy energetic nuclei are sources of strong electric and magnetic fields, it is of interest to ask whether such fields can have an effect on the formation of DCC. Some such effects are to be expected because electromagnetic fields break isospin symmetry, which is a subgroup of the $SU(2)_L \times SU(2)_R$ group of chiral isospin transformations. Expressed differently, electromagnetic fields affect the charged components of the order parameter $\vec{\phi} = (\sigma, \vec{\pi})$ of the QCD vacuum.

In this paper we discuss the effect of strong electromagnetic fields on chiral orientation in the framework of the linear σ model. Based on lessons we learn from the computation of the effective potential at one-loop level, we argue that the chiral $U(1)$ anomaly dominates the electromagnetic effects on DCC formation. We will then observe that the anomaly effect gives a quasi-instantaneous "kick" to the σ-model field configurations along π^0 direction in relativistic heavy ion collisions. We will attempt to estimate how effective a "kick" is in changing the chiral orientation of the DCC domain.

As a first step of this investigation we calculate the one-loop effective potential under the influence of an external static and uniform electromagnetic field in the framework of the linear σ model. The Lagrangian density of the model is given by

$$\mathcal{L} = \frac{1}{2} \sum D_\mu \vec{\phi} D^\mu \vec{\phi} - \frac{\lambda}{4} \left(|\vec{\phi}|^2 - f_\pi^2 \right)^2 + m_\pi^2 f_\pi \sigma \tag{1}$$

where $\vec{\phi} = (\sigma, \vec{\pi})$ and $D_\mu = \partial_\mu - ieA_\mu \hat{Q}$ denotes the covariant derivative with charge operator

$\hat{Q}$ of the chiral fields.

We use Schwinger's proper-time formalism [8] to compute the one-loop effective potential of the the chiral fields. Assuming that the charged fields do not develop nonvanishing vacuum values, we integrate over the charged field fluctuations (which are the only ones that couple to electromagnetic fields) around the charge-neutral π_3 and σ background fields. The result of a one-loop calculation for the effective potential can be represented as:

$$V_{\rm eff} = \frac{\lambda}{4}(|\vec{\phi}|^2 - f_\pi^2)^2 - f_\pi m_\pi^2 \sigma - \mathcal{L}'_{\rm em} \tag{2}$$

where $\mathcal{L}'_{\rm em}$ summarizes the one-loop effects and takes the form

$$\mathcal{L}'_{\rm em} = \frac{\alpha\lambda F^2}{24\pi m_\pi^2}(\sigma^2 + \pi_3^2 - f_\pi^2) + \frac{7\alpha^2 F^4}{360 m_\pi^4}, \tag{3}$$

for weak fields ($eF \ll m_\pi^2$; $F = \sqrt{E^2 - H^2}$), and

$$\mathcal{L}'_{\rm em} = \frac{\lambda|eF|}{16\pi^2}(\sigma^2 + \pi_3^2 - f_\pi^2) + \frac{\alpha F^2}{4\pi}\log\frac{eF}{m_\pi^2}. \tag{4}$$

for strong fields ($eF \gg m_\pi^2$).

It is easily seen from these expressions that the electromagnetic fields do not alter the orientation of the condensate $\langle\sigma\rangle$ in the ground state, a disappointing result. The reason for this result is very general; the chiral orientation remains unchanged because the electromagnetic effect leaves an SO(2) subgroup (σ, π_3) of the full SO(4) symmetry of the sigma model invariant. To show this, let us write the effective potential in a generic form

$$V_{\rm eff} = U(\sigma^2 + \pi_3^2) - f_\pi m_\pi^2 \sigma, \tag{5}$$

where U is an arbitrary function. It is then straightforward to show that the ground state of the potential is determined as

$$\sigma = \frac{f_\pi m_\pi^2}{2U'(\sigma^2)}, \qquad \pi_3 = 0. \tag{6}$$

We emphasize that this result is valid not only for static, uniform electromagnetic fields but also for any space-time dependent fields. This is due to the fact that the unbroken SO(2) symmetry survives under any such external electromagnetic fields.

We should note, however, that the SO(2) symmetry is to be broken if higher-loop corrections of the chiral fields are taken into account. We restrict ourselves to the one-loop order, the lowest nontrivial order having electromagnetic effects, because we regard the linear σ model as a low-energy effective theory of QCD. Therefore, we reach our first conclusion that the electromagnetism does not affect the chiral orientation of a DCC domain.

In the foregoing discussion we have ignored a fundamental property of underlying QCD, the existence of the chiral U(1) anomaly (Adler-Bell-Jackiw anomaly) [9]. It gives rise to an interaction term, the Wess-Zumino term, of the Nambu-Goldstone bosons with electromagnetic fields in the low-energy effective Lagrangian. The effective potential (4) is then modified into

$$V_{\rm eff} = U(\sigma^2 + \pi_3^2) - f_\pi m_\pi^2 \sigma - \frac{\alpha}{\pi f_\pi} \vec{E} \cdot \vec{H} \pi_3, \tag{7}$$

in which the $SO(2)$ symmetry is broken due to the Wess-Zumino term. The condition for the minimum in the variable σ remains unchanged from (7), but the minimum in the π_3 direction now occurs at the position

$$\frac{\pi_3}{\sigma} = \frac{\alpha}{\pi f_\pi^2 m_\pi^2} \vec{E} \cdot \vec{H}, \tag{8}$$

reflecting the relative strengths of the nonconservation of the axial $U(1)$ and the axial $SU(2)$ currents. Inserting the minimum position (8) back into the effective potential (7) we find

$$V_{\rm eff}^{\rm min} = U(\sigma^2 + \pi_3^2) - f_\pi m_\pi^2 \sigma - \left(\frac{\alpha \vec{E} \cdot \vec{H}}{\pi f_\pi m_\pi} \right)^2 \frac{\sigma}{f_\pi}. \tag{9}$$

This shows that the energy of the vacuum is always reduced by the anomaly contribution to the effective potential.

We first discuss what would be the general features of the effects of the anomaly driven term. Suppose that a DCC domain is formed during a heavy ion collision. How does the anomaly effect modify the properties of the coherent pion emission from DCC? It must be clear from (8) that the π^0 component is enhanced compared with that expected by the isospin symmetry; more anti-Centauro events should occur due to the chiral anomaly.

Let us clarify the meaning of this statement. If the isospin symmetry is preserved, we have, as the distribution of the neutral-to-all-pion ratio R

$$P(R) = \frac{1}{2}\,\frac{1}{\sqrt{R}}. \tag{10}$$

If we define the Centauro (anti-Centauro) events by $0 \leq R \leq \varepsilon$ ($1-\varepsilon < R \leq 1$) the ratio of number of events of anti-Centauro to Centauro is about $\sqrt{\varepsilon}/2$ for small ε. Our result implies that the anomaly effect increases this ratio. To quantify the size of this effect we have to estimate the magnitude of the anomaly term.

The anomaly term only contributes when parallel electric and magnetic fields are present. This is, indeed, generally the case in collisions of heavy nuclei, except in the extreme case of exactly head-on collisions. Of interest here are collisions at an impact parameter comparable to the nuclear radii, in which a significant fraction, say one-half, of the nucleons of both nuclei participate. Since only long-wavelength fluctuations of the chiral order parameter can serve as seeds for DCC, electromagnetic fields created by individual quarks, which dominate over short distance scales, are irrelevant, and only the coherent fields due to the nuclear charges need to be considered.

To get an understanding of the orders of magnitude involved, we can then estimate the electric and magnetic fields generated by fast moving nuclei by using the fields at their surface. In the center-of-mass (c.m.) system we have

$$E \approx H \approx \frac{Ze\gamma}{4\pi R^2}, \tag{11}$$

where Z and R are the nuclear charge and radius, respectively, and γ is the Lorentz factor in the c. m. system. When the nuclei are not too relativistic, so that they are not Lorentz contracted to less than the pion Compton wavelength ($R/\gamma > m_\pi^{-1}$), we can consider the change in the value of the electric and magnetic fields as quasi-adiabatic. Using (8) we find that the change in orientation in the ($\sigma - \pi_3$) plane is

$$\frac{\pi_3}{\sigma} = \left(\frac{Z\alpha\gamma}{2\pi f_\pi m_\pi R^2}\right)^2 \leq \left(\frac{Z\alpha}{2\pi f_\pi R}\right)^2 \approx 10^{-3} \tag{12}$$

for two colliding Pb or Au nuclei. Therefore, the effect does not appear to be sizable for nonrelativistic collisions.

On the other hand, if the nuclei are ultrarelativistic ($E/A \gg 10$ GeV/u), they are Lorentz contracted to pancakes far narrower than m_π^{-1} and the electromagnetic anomaly provide a quasi-instantaneous "kick" of the ground state away from the σ-axis. During the brief moment of overlap of the nuclei, the effective potential in the π_3 variable in the vicinity of the normal ground state has the form

$$V_{\rm eff}(\pi_3) = \frac{1}{2} m_\pi^2 \pi_3^2 - \left(\frac{Z\alpha\gamma}{2\pi f_\pi R^2} \right)^2 f_\pi \pi_3. \tag{13}$$

The linear term driving the π_3 field to nonzero values lasts only for a time of order R/γ, imparting a kick to the conjugate field momentum of order

$$\Delta \dot{\pi}_3 = \frac{(Z\alpha)^2 \gamma}{4\pi^2 f_\pi R^3} \approx \frac{1}{60} m_\pi^2 \tag{14}$$

for two colliding Au nuclei at RHIC ($\gamma = 100$). This results in a coherent excitation of the chiral order parameter in the π_3 direction over a volume of nuclear dimensions. Although the amplitude is quite small on the scale of m_π, its coherence over a region much larger than m_π^{-1} could be quite important because it establishes an explicit breaking of isospin symmetry in the initial conditions for the formation of a DCC state.

One can elaborate these estimations by computing the Lienard-Wiechert potential of the moving heavy ions under the point-nucleus approximation. One obtains

$$\vec{E} \cdot \vec{B} = -\frac{2Z^2 e^2}{M} \frac{\gamma^2}{R_1^3 R_2^3} (\vec{r} \cdot \vec{L}) \tag{15}$$

where $\vec{L} = \vec{b} \times M\vec{v}$ with ion mass M and impact parameter $\vec{b}$, and

$$R_{1,2} = \sqrt{\gamma^2 (z \mp vt)^2 + \left(\vec{r}_\perp \pm \frac{\vec{b}}{2} \right)^2}. \tag{16}$$

The origin of the coordinate system is taken at the collision point. The expression (15) indicates that the anomaly effect vanishes on the scattering plane and π_3 is kicked to the opposite direction in the northern and the southern hemispheres with respect to the axis

defined by $\vec{L}$. Using $|r| \sim |b| \sim R$ and $v \sim 1$ for the sake of an estimate, one obtains essentially the same expression as (13) except for some enhancement factor of order 10 in the vicinity of the ions.

Let us attempt a rough estimate of how the "kick" toward the π_3 direction affects DCC formation. For this purpose we follow the treatment by Blaizot and Krzywicki [10]. They analyze the classical field equations of the linear σ model in the 1+1 dimensional approximation. They solve the motion along the proper-time (τ) direction which may be appropriate for ultrarelativistic collisions, where initial conditions are expected to be approximately boost invariant [11]. In the reasonable limit $m_\sigma \gg m_\pi$ one can ignore the chiral symmetry breaking term. In fact, one can verify that coefficients of the quartic term in their two-dimensional Lagrangian are larger than m_π^2 by an order of magnitude. One then finds two constant vectors in isospin space (with prime denoting the derivative by τ):

$$\vec{a} = \tau(\vec{\pi} \times \vec{\pi}') \tag{17}$$

$$\vec{b} = \tau(\vec{\pi}\sigma' - \sigma\vec{\pi}') \tag{18}$$

corresponding to the vector and the axial-vector current conservation, respectively.

If the anomaly effect is included, the third component of the vector $\vec{b}$ is no longer conserved during the encounter of the two nuclei but obtains an additional contribution

$$\Delta b_3 = -\frac{\alpha}{\pi f_\pi^2} \int d\tau\, \tau(\vec{E} \cdot \vec{B})\sigma. \tag{19}$$

Therefore, the effect of the anomaly on b_3 vanishes at the chirally symmetric point $\sigma = 0$, and it is maximal at $\sigma = 1$. (Note that σ is rescaled by f_π in two dimensions.) By doing a similar estimate as we did earlier we obtain

$$\Delta b_3 \approx \frac{1}{8\pi^2} \left(\frac{Z\alpha}{f_\pi R}\right)^2 \approx 10^{-3} \tag{20}$$

for $\tau \sim R/\gamma$ and $f_\pi R \approx 3$. Thus, this estimate indicates that the anomaly effect is small at least numerically.

One might conclude that a kick of such tiny magnitude has no chance of affecting the DCC formation. However, we argue that it may well be effective in triggering the formation

of chirally misaligned domain. It is natural to suspect that at the pre-DCC stage collision debris is still hot and the chiral symmetry is restored. Therefore, the ground state is at around the top of the Mexican-hat potential. There are thermal fluctuations which trigger the rolling-down motion from the top. But they trigger the motion in a chirally symmetric way. There is the effect of the electromagnetic mass difference between charged and neutral pions, which may be comparable in magnitude — $(m_{\pi^+}^2 - m_{\pi^0}^2)/(m_{\pi^+}^2 + m_{\pi^0}^2) \sim 1/30$ — with the effect of the anomaly (14). However, the isospin structure of this term is quite different, $\Delta I = 2$ as opposed to $\Delta I = 1$ of the anomaly term, and it does not provide a force away from the axis $\langle \vec{\pi} \rangle = 0$. We emphasize that the direction of the "kick" is uniform over the volume of the collision debris. Since it has opposite directions in two hemispheres separated by collision plane we suspect that the effect may enhance the formation of two DCC domains. We therefore believe that the anomaly effect, in spite of its smallness, may play a role in the formation of DCC.

One of the best way of evaluating the anomaly effect on DCC formation is to implement its effect into the numerical simulations [4–6]. The characteristic features of the anomaly "kick" into the π_3 direction, such as "polarization" (i.e., the orientation change in sign from northern to southern hemispheres), may play an important role in incorporating the effect.

To summarize, we have discussed how strong electromagnetic fields affect chiral orientation within the framework of the linear σ model. We have found that the chiral U(1) anomaly of QCD, represented by the Wess-Zumino term in the low-energy effective Lagrangian, plays a crucial role. We have argued that it produces a "kick" to the field configurations along the π^0 direction over a volume of nuclear dimensions. We then have discussed the possibility that a small "kick" can act as an efficient trigger in the formation of the DCC domain. If this is the case the isospin breaking effect may enhance anti-Centauro over Centauro events in high energy nuclear collisions.

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